# Paper Reference(s) 66664/01 Edexcel GCE Core Mathematics C2 Bronze Level B4

## Time: 1 hour 30 minutes

Materials required for examination Mathematical Formulae (Green) Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

### **Instructions to Candidates**

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

### **Information for Candidates**

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. There are 9 questions in this question paper. The total mark for this paper is 75.

### **Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

### Suggested grade boundaries for this paper:

A*	Α	A B		D	Е	
73	67	61	54	47	40	

#### Using calculus, find the coordinates of the stationary point on the curve with equation 1.

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$
(6)
May 2013 (R)
$$f(x) = 2x^3 - 7x^2 - 10x + 24.$$
(a) Use the factor theorem to show that (x + 2) is a factor of f(x).
(2)

(b) Factorise f(x) completely.

# May 2012

3. Given that 
$$\begin{pmatrix} 40 \\ 4 \end{pmatrix} = \frac{40!}{4!b!}$$
,

2.

(a) write down the value of b.

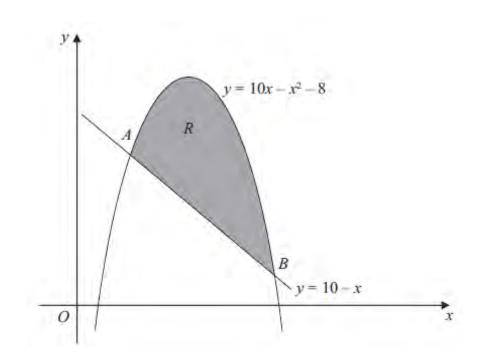
(1)

(4)

In the binomial expansion of  $(1 + x)^{40}$ , the coefficients of  $x^4$  and  $x^5$  are p and q respectively.

(b) Find the value of 
$$\frac{q}{p}$$
.

(3) January 2011



### Figure 1

Figure 1 shows the line with equation y = 10 - x and the curve with equation y = 10x - x<sup>2</sup> - 8.
The line and the curve intersect at the points A and B, and O is the origin.
(a) Calculate the coordinates of A and the coordinates of B.
(5)
The shaded area R is bounded by the line and the curve, as shown in Figure 1.
(b) Calculate the exact area of R.

5.	Given that	t 2 log2 (r +	$+15) - \log_2 x = 6$ ,
J.	UIVUII uia	$121022(\lambda +$	$1022 \lambda - 0$

(a) show that  $x^2 - 34x + 225 = 0$ .

(b) Hence, or otherwise, solve the equation  $2 \log_2(x+15) - \log_2 x = 6$ .

(2)

(5)

(7)

May 2012

January 2013

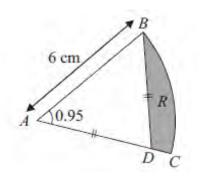




Figure 2 shows *ABC*, a sector of a circle of radius 6 cm with centre *A*. Given that the size of angle *BAC* is 0.95 radians, find

(a) the length of the arc $BC$ ,	
	(2)
(b) the area of the sector $ABC$ .	(2)
The point <i>D</i> lies on the line <i>AC</i> and is such that $AD = BD$ . The region <i>R</i> , shown Figure 2, is bounded by the lines <i>CD</i> , <i>DB</i> and the arc <i>BC</i> .	shaded in
(c) Show that the length of $AD$ is 5.16 cm to 3 significant figures.	
	(2)
Find	
(d) the perimeter of $R$ ,	
	(2)
(e) the area of $R$ , giving your answer to 2 significant figures.	(4)
	(+)

January 2012

7. (*a*) Show that the equation

8.

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0.$$
 (2)

(*b*) Hence solve, for  $0 \le x < 360^\circ$ ,

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

January 2011

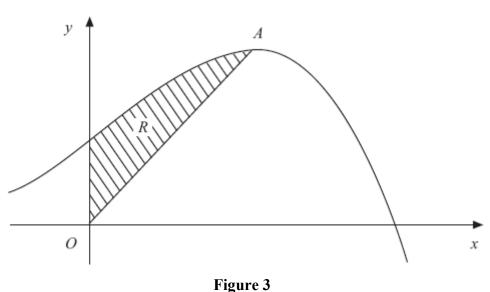


Figure 3

Figure 3 shows a sketch of part of the curve with equation  $y = 10 + 8x + x^2 - x^3$ .

The curve has a maximum turning point *A*.

(*a*) Using calculus, show that the *x*-coordinate of *A* is 2.

The region R, shown shaded in Figure 3, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

(3)

June 2008

9. The first three terms of a geometric series are (k + 4), k and (2k - 15) respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$ .	
	(4)
(b) Hence show that $k = 12$ .	(2)
(c) Find the common ratio of this series.	(-)
	(2)
( <i>d</i> ) Find the sum to infinity of this series.	
	(2)
	January 2009

## **TOTAL FOR PAPER: 75 MARKS**

END

Question number	Scheme	Marks
1	$\frac{dy}{dx} = 2 - 16x^{-3}$	M1 A1
	dx $2-16x^{-3} = 0$ so $x^{-3} = 0$ or $x^{3} = 0$ or $2-16x^{-3} = 0$ so $x = 2$	M1
	x = 2 only (after correct derivative)	A1
	$y = 2 \times "2" + 3 + \frac{8}{"2^2}"$	M1
	= 9	A1
		[6]
2 (a)	$f(-2) = 2.(-2)^{3} - 7.(-2)^{2} - 10.(-2) + 24$	M1
	= 0 so $(x+2)$ is a factor	A1
		(2)
(b)	$f(x) = (x+2)(2x^2 - 11x + 12)$	M1 A1
	f(x) = (x+2)(2x-3)(x-4)	dM1 A1
		(4)
		[6]
<b>3</b> (a)	$\binom{40}{4} = \frac{40!}{4!b!}$ ; $(1+x)^n$ coefficients of $x^4$ and $x^5$ are p and q respectively.	
	<i>b</i> = 36	B1
	Candidates should usually "identify" two terms as their $p$ and $q$	
	respectively.	
		(1)
(b)	Term 1:	
	$\begin{pmatrix} 40 \\ 4 \end{pmatrix}$ or ${}^{40}C_4$ or $\frac{40!}{4!36!}$ or $\frac{40(39)(38)(37)}{4!}$ or 91390	
	Any one of Term 1 or Term2 correct. (Ignore the label of $p$ and/or $q$ .)	M1
	Term 2:	
	$\begin{pmatrix} 40 \\ 5 \end{pmatrix}$ or ${}^{40}C_5$ or $\frac{40!}{5!35!}$ or $\frac{40(39)(38)(37)(36)}{5!}$ or 658008	
	Both of them correct. (Ignore the label of $p$ and/or $q$ .)	A1
	Hence, $\frac{q}{p} = \frac{658008}{91390} \left\{ = \frac{36}{5} = 7.2 \right\}$ for $\frac{658008}{91390}$ oe	A1 oe cso
		(3)
		[4]

Question number	Scheme					
4 (a)	Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic	Or puts $y = 10(10 - y) - (10 - y)^2 - 8$ and rearranges to give three term quadratic	M1			
	Solves their " $x^2 - 11x + 18 = 0$ " using acceptable method as in general principles to give $x =$	Solves their " $y^2 - 9y + 8 = 0$ " using acceptable method as in general principles to give $y =$	M1			
	Obtains $x = 2$ , $x = 9$ (may be on diagram or in part (b) in limits)	Obtains $y = 8$ , $y = 1$ (may be on diagram)	A1			
	Substitutes their x into a given equation to give $y =$ (may be on diagram)	Substitutes their y into a given equation to give $x = (may be on diagram or in part (b))$	M1			
	y = 8, y = 1	x = 2, x = 9	A1			
			(5)			
(b)	$\int (10x - x^2 - 8) dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \left\{-\frac{10x^2}{2} - \frac{x^3}{3} - 8x\right\}$	$+ c$ }	M1A1A1			
	$\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x\right]_2^9 = (\dots) - (\dots)$	dM1				
	$=90 - \frac{4}{3} = 88\frac{2}{3} \text{ or } \frac{266}{3}$					
	Area of trapezium = $\frac{1}{2}(8+1)(9-2) = 3$	B1				
	So area of <i>R</i> is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{3}{2}$	M1A1 cao (7)				
			[12]			

Question number	Sch	eme	Marks
5 (a)	$2\log(x+15) = \log(x+15)^2$		B1
	$\log(x+15)^{2} - \log x = \log \frac{(x+15)^{2}}{x}$ $2^{6} = 64 \text{ or } \log_{2} 64 = 6$	Correct use of $\log a - \log b = \log \frac{a}{b}$	M1
	$2^6 = 64 \text{ or } \log_2 64 = 6$	64 used in the correct context	B1
	$\log_2 \frac{(x+15)^2}{x} = 6 \Rightarrow \frac{(x+15)^2}{x} = 64$	Removes logs correctly	M1
	$\Rightarrow x^2 + 30x + 225 = 64x$ or x + 30 + 225x <sup>-1</sup> = 64	Must see expansion of $(x+15)^2$	
	$rac{}{} rac{}{} rx^{2} - 34x + 225 = 0 *$	to score the final mark.	A1
	$\dots x = J \pi + 22J = 0$		(5)
(b)	$(x-25)(x-9) = 0 \Longrightarrow x = 25 \text{ or } x = 9$	M1: Correct attempt to solve the <b>given</b> quadratic as far as $x =$	M1 A1
		A1: Both 25 and 9	
			(2)
			[7]
6 (a)	$r\theta = 6 \times 0.95, = 5.7$ (cm)	M1, A1	
		(2)	
(b)	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times 0.95, = 17.1 \text{ (cm}^2)$		M1, A1
			(2)
(c)	Let $AD = x$ then $\frac{x}{\sin 0.95} = \frac{6}{\sin 1.24}$		
	OR $x = 3 / \cos 0.95$ OR so $x = 3$	M1 A1	
	OR $x^2 = 6^2 + x^2 - 12x\cos 0.95$ leadi		
			(2)
(d)	Perimeter = '5.7'+5.16 +6 - 5.16= '	"11.7" or 6 + their 5.7	M1A1 ft
			(2)
(e)	Area of triangle $ABD = \frac{1}{2} \times 6 \times 5.16$		
	$\frac{1}{2} \times 6 \times 3 \times \tan 0.95 = 12.6$ ( <sup>1</sup> / <sub>2</sub> base x	height) or	
	$\frac{1}{2} \times 5.16 \times 5.16 \times \sin 1.24 = 12.6$		M1 A1
	So Area of $R = `17.1' - `12.6' = 4.5'$	5	M1 A1
			(4)
			[12]

Question number	Scheme							
7 (a)	$3\sin^2 x + 7\sin x = \cos^2 x - 4; \ 0 \le x < 360^\circ$							
	$3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4$							
	$4\sin^2 x + 7\sin x + 3 = 0  \mathbf{AG}$							
(b)	$(4\sin x + 3)(\sin x + 1) \{= 0\}$ Valid attempt at factorisation and $\sin x =$							
	$\sin x = -\frac{3}{4}$ , $\sin x = -1$ Both $\sin x = -\frac{3}{4}$ and $\sin x = -1$ .	A1						
	$( \alpha  = 48.59)$							
	$x = 180 + 48.59$ or $x = 360 - 48.59$ Either $(180 +  \alpha )$ or $(360 -  \alpha )$	dM1						
	x = 228.59, x = 311.41 Both awrt 228.6 and awrt 311.4	A1						
	$\{\sin x = -1\} \implies x = 270 $ <sup>270</sup>	B1						
		(5)						
		[7]						
<b>8</b> (a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 8 + 2x - 3x^2$							
	$3x^{2} - 2x - 8 = 0  (3x + 4)(x - 2) = 0  x = 2$							
(b)	Area of triangle = $\frac{1}{2} \times 2 \times 22$							
	$\int 10 + 8x + x^2 - x^3  dx = 10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$	M1 A1						
	$\int 10 + 8x + x - x  dx = 10x + \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$	A1						
	$\left[10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 = \dots \left(=20 + 16 + \frac{8}{3} - 4\right)$	M1						
	Area of $R = 34\frac{2}{3} - 22 = \frac{38}{3} \left( = 12\frac{2}{3} \right)$ (Or 12.6)	M1 A1						
		(8)						
		[11]						

number	Scheme					
9 (a)	Initial step: Two of: $a = k + 4$ , $ar = k$ , $ar^2 = 2k - 15$ Or one of: $r = \frac{k}{k+4}$ , $r = \frac{2k - 15}{k}$ , $r^2 = \frac{2k - 15}{k+4}$ , Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$ $k^2 = (k+4)(2k-15)$ , so $k^2 = 2k^2 + 8k - 15k - 60$	M1 M1, A1				
(b)	Proceed to $k^2 - 7k - 60 = 0$ (*) (k - 12)(k + 5) = 0 $k = 12$ (*)	A1 (4) M1 A1 (2)				
(c)	Common ratio: $\frac{k}{k+4}$ or $\frac{2k-15}{k} = \frac{12}{16} \left( = \frac{3}{4} \text{ or } 0.75 \right)$	M1 A1 (2)				
(d)	$\frac{a}{1-r} = \frac{16}{\binom{1}{4}} = 64$	M1 A1 (2) [10]				

## **Examiner reports**

## Question 1

The vast majority of candidates could differentiate the given function correctly, though a small number got  $x^{-1}$  rather than  $x^{-3}$ . Almost all candidates set the derivative equal to zero and found *x*, but a minority concluded  $x = \pm 2$ , ignoring the domain x > 0.

A small number forgot to find the *y* coordinate. Some candidates continued to find the second derivative here although this was unnecessary extra work.

## Question 2

This question was very well done and almost 75% of candidates achieved full marks or lost just one mark.

Use of the factor theorem was well understood in part (a). Many, having shown f(-2) = 0, lost the accuracy mark by not giving a conclusion such as 'therefore (x + 2) is a factor'. A few used long division in part (a) and gained no marks, as the question explicitly asked for the use of the factor theorem.

In part (b), achieving the full factorised expression for f(x) was very well done, but a few slipped up on the (2x - 3) factor, or thought x = -2, x = 4, x = 1.5 was the answer to the question. The distinction between solve and factorise should be understood by candidates at this level, but frequently is not.

## Question 3

A significant number of candidates failed to answer part (a) correctly, due to the unfamiliarity with the formula for  $\binom{n}{r}$ . Common incorrect answers for *b* included 1, 4, 10, 36! or 91390.

In part (b), most candidates were able to write down the binomial expansion of  $(1 + x)^n$ . Although a minority of candidates picked out wrong terms, most commonly terms in  $x^3$  and  $x^4$  rather than  $x^4$  and  $x^5$ , the majority of candidates were able to to give  $\frac{q}{p}$  as  $\frac{36}{5}$ . Other common errors included finding  $\frac{p}{q}$  or giving  $\frac{q}{p}$  as 7.2*x*, which is not independent of *x*.

## **Question 4**

In part (a) most candidates recognized that they needed to equate the line and curve equations and in most cases a correct quadratic equation and correct x-values were found. A few lost the next 2 marks by not deriving the corresponding y-values. Poor algebra was seen however and the incorrect  $x^2 - 9x + 18 = 0$  appeared regularly.

Part (b) saw separate integration of the curve and line equations, with use of the limits 2 and 9, proved a more successful approach than trying to combine the curve and line equations first, though stronger candidates had no problem. Sign errors were not uncommon by others who attempted to combine. Integration overall was very good, though some stopped after finding the area under the curve, not realizing that the area of the trapezium had to be subtracted. Geometric attempts at splitting the trapezium to obtain its area were often flawed, with the wrong formulae used. Others only subtracted a triangle instead of a trapezium. A sizeable minority found the points where the curve crossed the *x*-axis and used these values in their limits. This was unnecessary and frequently led to errors. The most common error was in not appreciating what an exact answer means, and rounded decimal answers were often seen and lost the final mark. Overall however this was an accessible question, and while 37% achieved full marks, 72% achieved 9 or more marks out of 12.

## **Question 5**

In Q5(a) logarithms were challenging for the less able candidates. Although many could apply the power rule correctly to obtain  $2\log (x + 15) = \log (x + 15)^2$ , some then proceeded to  $\frac{\log(x+15)^2}{\log x} = 6$ . Some candidates also erroneously started with  $\frac{2\log(x+15)}{\log x} = 6$  or  $2\log(\frac{x+15}{x}) = 6$  and were unable to gain much credit. The next stage was answered better

and many candidates knew that to remove logs,  $2^6$  was required on the right hand side.

Q5(b) involved solving the quadratic from Q5(a) and the majority opted to use factorisation successfully. Some chose to use the quadratic formula and were less successful, making arithmetic errors or using an incorrect formula.

## Question 6

Most candidates were able to display their knowledge of trigonometry and circles here and a substantial group (44%) achieved full marks. Among the others, greater clarity in their responses would have helped their own working and not led to lost marks. A few were reluctant to work in radians, particularly in part (c), but generally worked correctly in degrees.

Parts (a) and (b) were almost always correct, though some candidates lost the  $\frac{1}{2}$  in the formula for area.

Part (c) was attempted in a wide variety of ways. The most common approach was to use the Sine rule, having first found that the third angle of triangle ADB is 1.24 radians. This was generally successful. However there were a few cases seen of angle  $(2\pi - 2x) \times 0.95$  and there was some confusion as to which were the equal angles of the isosceles triangle.

Others used trigonometry in the right-angled triangle which is half of triangle *ADB*, getting a successful result from  $AD = \frac{3}{\cos 0.95}$ . Those who attempted the cosine rule in triangle *ADB* could achieve a correct answer but sometimes attempted a verification method. Answers to part (c) were often disappointing, partly from poor algebra and from a lack of clarity in the symbols used and confusion about the equal sides. As this was a 'show that' question, there appeared to be a temptation for some to hope that the examiner would not notice incorrect working.

In part (d), where perimeter was needed, a few slips were seen, but most were able to achieve the required result. Many were able to find the correct area in part (e) by using the difference between the area of the sector and of the triangle ADB. Lack of clarity was a problem where errors occurred, since scripts mostly said 'the area of the triangle' and it was not always clear whether ADB or ABC was intended. Some approached the problem by using area of the segment + area of triangle BDC. This was usually successful. Errors sometimes occurred when finding angles in triangle BDC. Lack of clarity again caused difficulties. A minority of candidates assumed that this was a normal 'area of segment' question, without looking properly at the question, but this was fairly rare.

## **Question 7**

Most candidates were able to score both marks in part (a). Most candidates proceeded by replacing  $1 - \sin^2 x$  for  $\cos^2 x$ . A few candidates, however, made algebraic errors or slips in rearranging the equation correctly into the result given.

The need to use the alternative form was understood in part (b) and most candidates made a valid attempt at factorisation, with correct factors being seen much more frequently than incorrect ones. Some candidates correctly wrote  $(4\sin x + 3)(\sin x + 1) = 0$  and solved this incorrectly to give one of their solutions as  $\sin x = \frac{3}{4}$ . Of those candidates achieving the correct two values for  $\sin x$  many only gave two correct solutions, usually 228.6 and 270 or 311.4 and 270. Sometimes extra incorrect solutions were given, usually 131.4 and/or 90. A small number of candidates found  $(270 \pm \text{their } |\alpha|)$  rather than  $(180 + |\alpha|)$  and  $(360 - |\alpha|)$ . Some candidates incorrectly stated that  $\sin x - 1$  had no solutions and a few gave their answers to the nearest degree. A significant number of candidates used a sketch of  $\sin x$  to help them to correctly identify their answers.

### **Question 8**

In general, this was very well done, showing that many candidates know when to differentiate and when to integrate.

In part (a), most candidates successfully differentiated the equation then presented evidence that they understood that  $\frac{dy}{dx} = 0$  at a turning point. A significant number went on to find the second derivative to establish the nature of the turning point (although this was not required here, given the graph).

### Question 9

Part (a) was a good discriminator. There were a few cases of "fudging" attempts to yield the printed answer using (k + 4)(2k - 15) = 0 or similar. Cancelling was often ignored by those using  $(k + 4) \ge (k/(k + 4))^2 = (2k - 15)$  resulting in cubic equations – generally incorrectly expanded.

Finding the printed answer in (b) was straightforward and most were successful at solving the quadratic equation. Some used verification and lost a mark.

Finding the common ratio in part (c) was answered well, though some candidates found r = 4/3 however.

The sum to infinity in (d) was answered well. Using 12 for "a" was the frequent error here.

			Mean score for students achieving grade:								
Qu	Max score	Modal score	Mean %	ALL	<b>A</b> *	Α	В	С	D	Е	U
1	6		86	5.17	5.93	5.84	5.56	5.04	4.44	3.96	2.24
2	6		81	4.86	5.84	5.68	5.44	5.18	4.86	4.30	2.72
3	4		75	2.99	3.80	3.59	3.19	2.87	2.59	2.13	1.16
4	12		77	9.18	11.87	11.50	10.79	10.03	9.07	7.50	3.47
5	7	7	78	5.47	7.00	6.76	6.17	5.25	4.29	3.63	1.89
6	12		77	9.26	11.53	11.31	10.29	9.00	7.44	5.87	3.42
7	7		78	5.43	6.93	6.59	6.02	5.28	4.27	3.33	1.92
8	11		68	7.46		10.62	9.57	8.15	6.11	4.04	1.32
9	10		65	6.54		9.04	7.18	5.71	4.50	3.47	1.96
	75		75.15	56.36	52.90	70.93	64.21	56.51	47.57	38.23	20.10

## Statistics for C2 Practice Paper Bronze Level B4